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SOLUTIONS OF PROBLEMS IN NUMBER TWO.

SOLUTIONS of problems in No. 2 have been received as follows:

From Prof. L. G. Barbour, 340; Prof. W. P. Casey, 340, 341; G. E. Curtis, 339, 340; Prof. E. J. Edmunds, 340; Geo Eastwood, 340, 345; Prof. A. B. Evans, 340; W. E. Heal, 340; Prof. E. W. Hyde, 343; Wm. Hoover, 339, 340; Prof. D. J. Mc Adam, 340; C. H. Metcalf, 340; Prof. E. B. Seitz, 344; E. Vansickel, 340; R. S. Woodward, 339, 340, 345.

339. "Required the average distance from the center of a circle to all points in the surface of a sector."

SOLUTION BY G. E. CURTIS, YALE COLLEGE, CONN.

Let the arc of the sector be denoted by a and the distance from the centre to any point of the surface by ρ . Then the sum of the distances from the center to all points of the sector will be

$$\int_0^a \int_0^r \rho \cdot \rho d\rho d\theta = \frac{1}{3} r^3 a$$

Hence the average distance is $\frac{1}{3} r^3 a \div \frac{1}{2} r^2 a = \frac{2}{3} r$

340. "Integrate $\frac{dx}{\sin x + \cos x}$."

SOLUTION BY PROF. W. P. CASEY, SAN FRANCISCO, CAL.

$$\int \frac{dx}{\sin x + \cos x} = \int \frac{\sin x dx}{\sin^2 x + \sin x \cos x} = \int \frac{dz}{1-z^2 + (1-z^2)^{\frac{1}{2}} z}, \text{ if } z = \cos x.$$

By integrating, reducing and restoring the value of z , we find

$$\int \frac{dx}{\sin x + \cos x} = \frac{1}{\sqrt{2}} \log \tan \left(\frac{x}{2} + \frac{\pi}{8} \right).$$

141. "Show that 'Every even number is the sum of two prime numbers, and every odd number is the sum of three prime numbers.' Barlow's Theory of Numbers, page 259."

SOLUTION BY PROF. E. J. EDMUNDS, SOUTH'N UNIV., N. ORLEANS, LA.

It is well known that every prime number is of the form $6x \pm 1$, x being any integer. Hence

$$6x+1+6x-1=12x$$

which is an even number. We have also, putting $x = n$,

$$2n+1 = 6x+1+6x-1+1 = \text{an odd number.}$$

342. No solution received.

343. "If E^2 be the sum of the squares of the edges of a tetrahedron, F^2 the sum of the squares of the areas of the faces and V the volume, show that the principal semi axes of the ellipsoid inscribed in the tetrahedron, touching each face in the center of gravity and having its center at the center of gravity of the tetrahedron, are the roots of

$$k^6 - \frac{E^2}{2^4 \cdot 3} k^4 + \frac{F^2}{2^4 \cdot 3^2} k^2 - \frac{V^2}{2^6 \cdot 3} = 0."$$

SOLUTION BY PROF. E. W. HYDE.

Let the equation of the ellipsoid be $S\rho\varphi\rho = 1$, in which

$$\varphi\rho = \alpha S\alpha'\rho + \beta S\beta'\rho + \gamma S\gamma'\rho.$$

Let α, β, γ and δ be the vectors from the center of gravity of the tetrahedron, taken as the origin, to its vertices; then $\delta = -(\alpha + \beta + \gamma)$.

The vector from the origin to the c. g. of face opposite α is $-\frac{1}{3}\alpha$, and the perpendicular on this face from the origin is

$$[\varphi(-\frac{1}{3}\alpha)]^{-1} = [V(\beta\gamma + \gamma\delta + \delta\beta)]^{-1} S\beta\gamma\delta = [V(\alpha\beta - 3\beta\gamma + \gamma\alpha)]^{-1} S\alpha\beta\gamma.$$

$$\therefore \varphi\alpha = 3V(3\beta\gamma - \gamma\alpha - \alpha\beta)S^{-1}\alpha\beta\gamma;$$

and similarly

$$\varphi\beta = 3V(3\gamma\alpha - \alpha\beta - \beta\gamma)S^{-1}\alpha\beta\gamma;$$

$$\varphi\gamma = 3V(3\alpha\beta - \beta\gamma - \gamma\alpha)S^{-1}\alpha\beta\gamma.$$

Now for the axes $\varphi\rho$ must coincide in direction with ρ ; \therefore make

$$\varphi\rho = -k^{-2}\rho, (\varphi + k^{-2})\rho = 0.$$

Operating by $S\rho$ this gives for the value of k , $k^2 = T^2\rho$. It may be easily shown that the discriminating cubic is

$$S(\varphi + k^{-2})\alpha(\varphi + k^{-2})\beta(\varphi + k^{-2})\gamma = 0;$$

or expanding

$$k^{-6} + \frac{S(\alpha\beta\varphi\gamma + \beta\gamma\varphi\alpha + \gamma\alpha\varphi\beta)}{S\alpha\beta\gamma} \cdot k^{-4} + \frac{S(\alpha\varphi\beta\varphi\gamma + \beta\varphi\gamma\varphi\alpha + \gamma\varphi\alpha\varphi\beta)}{S\alpha\beta\gamma} \cdot k^{-2} + \frac{S\varphi\alpha\varphi\beta\varphi\gamma}{S\alpha\beta\gamma} = 0.$$

The coefficient of k^{-4} gives on substituting its value for $\varphi\gamma$, etc.,

$$\begin{aligned} & \frac{3}{S^2\alpha\beta\gamma} \left[3(V^2\alpha\beta + V^2\beta\gamma + V^2\gamma\alpha) - 2(S\beta\gamma V\alpha\beta + S\gamma\alpha V\beta\gamma + S\alpha\beta V\gamma\alpha) \right] \\ = & \frac{3}{4S^2\alpha\beta\gamma} \left[V^2(\beta - \alpha)(\gamma - \alpha) + V^2(\beta - \alpha)(\delta - \alpha) + V^2(\delta - \alpha)(\gamma - \alpha) \right. \\ & \left. + V^2(\beta - \delta)(\gamma - \delta) \right] = \frac{-4F^2}{3V^3}. \end{aligned}$$

For the coefficient of k^{-2} we find

$$\begin{aligned} & -\frac{72}{S^2\alpha\beta\gamma} \cdot S(\alpha^2 + \beta^2 + \gamma^2 + \alpha\beta + \beta\gamma + \gamma\alpha) = -\frac{9}{S^2\alpha\beta\gamma} \left[(\alpha - \beta)^2 + (\beta - \gamma)^2 \right. \\ & \left. + (\gamma - \alpha)^2 + (\alpha - \delta)^2 + (\beta - \delta)^2 + (\gamma - \delta)^2 \right] = \frac{2^2 E^2}{V^2}. \end{aligned}$$

And finally

$$\frac{S\varphi a\varphi\beta\varphi\gamma}{S a\beta\gamma} = \frac{-2^4 \cdot 3^3}{S^2 a\beta\gamma} = -\frac{2^6 \cdot 3}{V^2}.$$

Substituting these values and multiplying through by $-V^2 k^6 \div 2^6 \cdot 3$ we have the required equation,

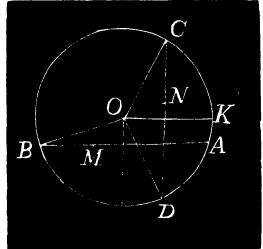
$$k^6 - \frac{E^2}{2^4 \cdot 3} k^4 + \frac{F^2}{2^4 \cdot 3^2} k^2 - \frac{V^2}{2^6 \cdot 3} = 0.$$

344. "Through each of two points, taken at random within a circle, a random chord is drawn; find (1) the prob'y that the chords will intersect; and (2) if a third random chord be drawn through a 3d random p't, find the probabilities that the 3 chords will intersect in 0, 1, 2, 3 points."

SOLUTION BY PROF. E. B. SEITZ.

1. Let M and N be two random points within the circle whose center is O , and AB , CD random chords drawn through them. Draw the radii OH , OK perpendicular to AB , CD .

Let $OA = r$, $AM = x$, $CN = y$, $AB = x'$, $CD = y'$, $\angle AOH = \theta$, $\angle COK = \varphi$, $\angle HOK = \mu$, and ω = the angle OH makes with a fixed radius. Then $x' = 2r \sin \theta$, $y' = 2r \sin \varphi$; an element of the circle at M is $r \sin \theta d\theta dx$, at N it is $r \sin \varphi d\varphi dy$, and for elemental changes in the directions of AB and CD we have $d\omega$ and $d\mu$.



The limits of θ are 0 and $\frac{1}{2}\pi$; of φ , 0 and θ , and θ and $\frac{1}{2}\pi$; of μ , $\theta - \varphi$ and $\theta + \varphi$, when $\varphi < \theta$, and $\varphi - \theta$ and $\varphi + \theta$, when $\varphi > \theta$, and the result doubled; of ω , 0 and 2π ; of x , 0 and x' ; and of y , 0 and y' . Hence, since the whole number of ways the two chords can be drawn is $\pi^4 r^4$, the requir'd prob'y is

$$\begin{aligned} p &= \frac{2}{\pi^4 r^4} \int_0^{\frac{1}{2}\pi} \int_0^{2\pi} \int_0^{x'} \left\{ \int_0^{y'} \left[\int_0^\theta \int_{\varphi-\theta}^{\varphi+\theta} r \sin \varphi d\varphi d\mu \right. \right. \\ &\quad \left. \left. + \int_\theta^{\frac{1}{2}\pi} \int_{\varphi-\theta}^{\varphi+\theta} r \sin \varphi d\varphi d\mu \right] dy \right\} r \sin \theta d\theta d\omega dx \\ &= \frac{1}{3} + \frac{5}{2\pi^2}. \end{aligned}$$

[Our space will not permit the insertion of the integration in detail as given by Prof. Seitz, nor of the second part, at present, but we will insert the analysis of the the second part in a future number.]

345. "The great circle from $A (\varphi_1, \lambda_1)$ to $B (\varphi_2, \lambda_2)$ passes north of the parallel of latitude φ_0 ; what is the longitude λ_0 of the point P on this parallel so that the course APB shall be the shortest course from A to B which does not pass north of this parallel?"

SOLUTION BY R. S. WOODWARD, DETROIT, MICH.

Since the course APB is to be a minimum, AP and PB must be arcs of great circles. Designate them by s_1 and s_2 respectively. Then

$$\cos s_1 = \sin \varphi_1 \sin \varphi_0 + \cos \varphi_1 \cos \varphi_0 \cos (\lambda_0 - \lambda_1) \quad (1)$$

$$\cos s_2 = \sin \varphi_2 \sin \varphi_0 + \cos \varphi_2 \cos \varphi_1 \cos (\lambda_2 - \lambda_0) \quad (2)$$

Since $(s_1 + s_2)$ is to be a minimum with respect to λ_0 , (1) and (2) give

$$\frac{d(s_1 + s_2)}{d\lambda_0} = \frac{\cos \varphi_1 \cos \varphi_0 \sin (\lambda_0 - \lambda_1)}{\sin s_1} - \frac{\cos \varphi_2 \cos \varphi_0 \sin (\lambda_2 - \lambda_1)}{\sin s_2} = 0,$$

whence

$$\frac{\cos \varphi_1 \sin (\lambda_0 - \lambda_1)}{\cos \varphi_2 \sin (\lambda_2 - \lambda_0)} = \frac{\sin s_1}{\sin s_2}. \quad (3)$$

Call the angles at P between PA and the meridian of λ_0 and between the latter and PB , θ_1 and θ_2 respectively; then

$$\frac{\cos \varphi_1 \sin (\lambda_0 - \lambda_1)}{\cos \varphi_2 \sin (\lambda_2 - \lambda_1)} = \frac{\sin s_1 \sin \theta_1}{\sin s_2 \sin \theta_2}; \quad (4)$$

(3) and (4) give

$$\sin \theta_1 = \sin \theta_2 \text{ or}$$

$$\theta_1 = \theta_2 \text{ or } (180^\circ - \theta_2).$$

Now the two spherical triangles whose common side is $(90^\circ - \varphi_0)$ give

$$\begin{aligned} \cot \theta_1 &= \frac{\tan \varphi_1 \cos \varphi_1 - \sin \varphi_0 \cos (\lambda_0 - \lambda_1)}{\sin (\lambda_0 - \lambda_1)} \\ &= \pm \cot \theta_2 = \pm \frac{\tan \varphi_2 \cos \varphi_0 - \sin \varphi_0 \cos (\lambda_2 - \lambda_0)}{\sin (\lambda_2 - \lambda_0)}, \end{aligned}$$

from which λ_0 may be readily found.

[Mr. George Eastwood has given an extended discussion of, and demonstration of the affirmation contained in, the Query at the foot of p. 31, but the space at our command will not permit its publication in this number; we hope, however to be able to present it to our readers in a future number.

We take this opportunity to notify our readers that, in consequence of our intended absence from home during the fore part of June, No. 4 will probably not be mailed to subscribers until about the 10th of July —Ed.]